NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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METHOD FOR RAPID DETERMINATION OF PRESSURE CHANGE FOR ONE-DIMENSIONAL FLOW WITH HEAT TRANSFER, FRICTION, ROTATION, AND AREA CHANGE

By James E. Hubbartt, Henry O. Slone, and Vernon L. Arne

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

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SUMMARY

An approximate method for rapid determination of the pressure change for subsonic flow of a compressible fluid under the simultaneous action of heat transfer, friction, rotation, and area change is developed. In the development of this method, the momentum equation was approximated and rearranged for a convenient solution employing charts. This report presents both the analysis involved in simplifying the momentum equation and the charts necessary for obtaining particular solutions. The charts provide a step-by-step solution which converges to an exact solution as the number of steps is increased. An illustrative example and comparison with more rigorous numerical solutions with conditions typical for air-cooled turbine blades are included. These comparisons show that the solution converges rapidly to provide good accuracy.

INTRODUCTION

The effective design of ducts to accommodate air flow requires particular solutions of the momentum equation for determining the pressure changes encountered. Simplified methods which give an accuracy sufficient for engineering purposes are in demand. An approximate method for determining such particular solutions has been developed for the one-dimensional flow of a compressible fluid under the influence of heat transfer, friction, rotation, and area change.

A number of studies of one-dimensional flow of a compressible fluid have been published. Reference 1 presents a rather complete treatment including a form of the momentum equation with heat transfer, friction, and area change which is suitable for numerical integration. As an aid for expediting the numerical integration, coefficients of the differentials in the momentum equation of reference 1 have been tabulated as "influence" coefficients in reference 2. The analyses of references 1 and 2 have been extended in reference 3 to include rotational forces.

Reference 3 also provides a form of the momentun equation suitable for numerical integration and tabulates "influence" coefficients for expediting the calculations. However, in all cases, the numerical integrations as employed in references 1 to 3 are tedious and time consuming. Particular solutions suitable for rapid determination of pressure changes for one-dimensional flow with both arbitrary and specific heat-input distributions have been presented in references 4 to 7. However, these solutions include only heat transfer and friction. In addition, reference 8 developed an approximate solution for constant flow area including rotation as well as heat transfer and friction. The approximations of reference 8 were based upon experimental data for cooled turbine blades tested under static conditions. Although the time required for a particular solution of the momentum equation and subsequent evaluation of the pressure change was appreciably reduced when the approximate solution of reference 8 was utilized, the time required for a large number of particular solutions, as frequently required for design procedures, remains very lengthy. In addition, reference 8 does not account for flow-area changes frequently encountered in actual practice.

Since a method for the rapid determination of the pressure change through ducts with heat transfer, friction, rotation, and area change is in demand, the momentum equation has been approximated and rearranged for a convenient solution employing charts. The purpose of the present report is to present the analysis involved in simplifying the momentum equation so that a graphical solution is possible and to present the charts necessary for obtaining particular solutions for subsonic flow. Heat transfer, friction, rotation, and area changes are all included in the solutions. The charts provide a step-by-step solution which converges to an exact solution as the number of steps is increased and is limited only by the graphical accuracy. Comparisons are also made with the solutions determined from the more exact numerical integration of reference 3 for typical examples of air-cooled turbine blades.

ANALYSIS

The general momentum equation for one-dimensional flow is approximated and rearranged to form the basis for the construction of charts suitable for the determination of the Mach number change with flow under the simultaneous action of heat transfer, friction, rotation, and area change. A relation between the Mach numbers and pressures is finally determined such that charts may be employed to evaluate the pressure changes in a duct once the Mach number changes are known. The energy changes may be arbitrarily specified in terms of the total-temperature changes and may be evaluated by a procedure such as that given in reference 3.

Evaluation of Mach Numbers

The differential form of the general momentum equation for one-dimensional flow is (ref. 3)

$$dP - F_b \rho g \, dx + \rho V \, dV + \frac{dF_d}{A} = 0 \tag{1}$$

where x increases in the flow direction (all symbols are defined in appendix A). The differential drag force can be expressed by the equation

$$dF_d = f \frac{\rho V^2}{2} \, dS = f \frac{\rho V^2}{2} \, l \, dx = \frac{4f}{D_h} \frac{\rho V^2}{2} A \, dx \tag{2}$$

and the body forces per pound of weight flow, as resulting from rotation about a central axis, can be expressed by the equation

$$F_b = \frac{\omega^2 r}{g} \tag{3}$$

If the differential momentum change is replaced by the equivalent expression

$$\rho V \, dV = d(\rho V^2) - V \, d(\rho V) \tag{4}$$

and equations (2) to (4) are substituted into equation (1), the momentum equation, as integrated between stations x and $x+\Delta x$, becomes

$$\int_{P_x}^{P_{x+\Delta x}} dP + \int_{(\rho V^2)_x}^{(\rho V^2)_{x+\Delta x}} d(\rho V^2) - \int_{(\rho V)_x}^{(\rho V)_{x+\Delta x}} V \, d(\rho V) -$$

$$\int_x^{x+\Delta x} \omega^2 r \rho \, dx + \int_x^{x+\Delta x} \frac{4f}{D_h} \frac{\rho V^2}{2} \, dx = 0 \tag{5}$$

The first two terms of equation (5) can be integrated directly. However, the last three integrals of equation (5) cannot be evaluated directly and therefore are approximated in order to obtain forms which can be conveniently solved. The third integral, which exists only in the case of area change, and the fifth integral, which expresses the

drag forces, may be conveniently approximated by using the integrand as the average of the values at the end points. That is,

$$\int_{(\rho V)_x}^{(\rho V)_{x+\Delta x}} V \, d(\rho V) \approx \frac{(V_x + V_{x+\Delta x})}{2} \left[(\rho V)_{x+\Delta x} - (\rho V)_x\right] \tag{6}$$

and

$$\int_x^{x+\Delta x} \frac{4f}{D_h} \frac{\rho V^2}{2} \, dx \approx \frac{\Delta x}{2} \left[\frac{4f_{x+\Delta x}}{D_{h,x+\Delta x}} \left(\frac{\rho V^2}{2}\right)_{x+\Delta x} + \frac{4f_x}{D_{h,x}} \left(\frac{\rho V^2}{2}\right)_x\right] \tag{7}$$

In order to facilitate a convenient solution, the fourth integral, which expresses the rotational forces in equation (5), will be approximated by either

$$\int_x^{x+\Delta x} \omega^2 r \rho \, dx \approx \omega^2 r_x \rho_x \Delta x \tag{8a}$$

or

$$\int_x^{x+\Delta x} \omega^2 r \rho \, dx \approx \omega^2 r_{x+\Delta x} \rho_{x+\Delta x} \Delta x \tag{8b}$$

depending upon whether the solution is initiated with the conditions at the station x or the station $x+\Delta x$. Since the integrand of either equation (8a) or (8b) is approximated by the value at a single end point, the approximation is in general less accurate than those given by equations (6) and (7). Equations (6) to (8) become identities in the limit as Δx approaches zero.

Because the density and velocity variations from stations x to $x+\Delta x$ are approximated by equations (6), (7), and (8), the energy equation must be employed only to determine the total energy change.

When the approximations given by equations (6), (7), and (8a) are substituted into equation (5) and the first two terms of equation (5) are integrated, the approximate momentum equation becomes

$$P_{x+\Delta x} - P_x + (\rho V^2)_{x+\Delta x} - (\rho V^2)_x - \frac{V_x + V_{x+\Delta x}}{2}\left[(\rho V)_{x+\Delta x} - (\rho V)_x\right] -$$

$$\omega^2 r_x \rho_x \Delta x + \frac{\Delta x}{2}\left[\frac{4f_{x+\Delta x}}{D_{h,x+\Delta x}}\left(\frac{\rho V^2}{2}\right)_{x+\Delta x} + \frac{4f_x}{D_{h,x}}\left(\frac{\rho V^2}{2}\right)_x\right] = 0 \qquad (9a)$$

However, if equation (8b) is used instead of equation (8a), the approximate momentum equation becomes

$$P_{x+\Delta x} - P_x + (\rho V^2)_{x+\Delta x} - (\rho V^2)_x - \frac{V_x + V_{x+\Delta x}}{2}\left[(\rho V)_{x+\Delta x} - (\rho V)_x\right] -$$

$$\omega^2 r_{x+\Delta x} \rho_{x+\Delta x} \Delta x + \frac{\Delta x}{2}\left[\frac{4f_{x+\Delta x}}{D_{h,x+\Delta x}}\left(\frac{\rho V^2}{2}\right)_{x+\Delta x} + \frac{4f_x}{D_{h,x}}\left(\frac{\rho V^2}{2}\right)_x\right] = 0 \qquad (9b)$$

If the variables of equations (9a) and (9b) are changed by employing the continuity equation, the relation between the total and static temperatures, and the equation of state, the two forms of the approximate momentum equation can be reduced to the useful relations (appendix B)

$$F(M_{x+\Delta x}, A_{x+\Delta x}) = \sqrt{\frac{T'_x}{T'_{x+\Delta x}}}\left(\frac{A_{x+\Delta x}}{A_x}\right)\left[F(M_x, A_x) + \frac{\omega^2 r_x \Delta x}{T'_x} G(M_x)\right] \qquad (10a)$$

and

$$F(M_x, A_x) = \sqrt{\frac{T'_{x+\Delta x}}{T'_x}}\left(\frac{A_x}{A_{x+\Delta x}}\right)\left[F(M_{x+\Delta x}, A_{x+\Delta x}) - \frac{\omega^2 r_{x+\Delta x} \Delta x}{T'_{x+\Delta x}} G(M_{x+\Delta x})\right] \qquad (10b)$$

where

$$F(M,\Lambda) = \frac{1}{M\left(1 + \frac{\gamma-1}{2} M^2\right)^{1/2}}\left[1 - \frac{\gamma M^2}{4}(\Lambda - 4)\right] \qquad (11)$$

$$G(M) = \frac{\left(1 + \frac{\gamma-1}{2} M^2\right)^{1/2}}{gRM} \qquad (12)$$

$$\Lambda_x = \frac{4f_x\Delta x}{D_{h,x}} - 2\left(\frac{A_x}{A_{x+\Delta x}} - 1\right) \tag{13}$$

$$\Lambda_{x+\Delta x} = -\frac{4f_{x+\Delta x}\Delta x}{D_{h,x+\Delta x}} - 2\left(\frac{A_{x+\Delta x}}{A_x} - 1\right) \tag{14}$$

The total-temperature ratio in equations (10) may be evaluated from a solution of the energy equation such as presented in reference 3.

Evaluation of Pressures

By applying the continuity equation and the equation of state, the weight flow can be expressed as

$$w = \frac{P}{RT} VA = \frac{P\sqrt{\gamma gR}}{R\sqrt{T}} MA \tag{15}$$

or

$$w = \frac{PA}{\sqrt{T'}}\sqrt{\frac{\gamma g}{R}}\, M\left(\frac{T'}{T}\right)^{1/2} \tag{16}$$

Since the relation between the total and static temperature as given by the energy equation is

$$\frac{T'}{T} = 1 + \frac{\gamma-1}{2} M^2 \tag{17}$$

equation (16) can be rewritten in the form

$$\frac{w\sqrt{T'}}{PA} = \sqrt{\frac{\gamma g}{R}}\, M\left(1 + \frac{\gamma-1}{2} M^2\right)^{1/2} \tag{18}$$

Using $P'/P = (T'/T)^{\gamma/(\gamma-1)}$ to express equation (18) in terms of the total pressure gives

$$\frac{w\sqrt{T'}}{P'A} = \sqrt{\frac{\gamma g}{R}}\, M\left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}} \tag{19}$$

Equations (18) and (19) are plotted in figure 1 for $\gamma = 1.40$ with $w\sqrt{T'}/PA$ and $w\sqrt{T'}/P'A$ as the ordinates and M as the abscissa. From the Mach number the value of either $w\sqrt{T'}/P'A$ or $w\sqrt{T'}/PA$ can be determined from figure 1. Then from the given conditions and area, P or P' can be computed.

CALCULATION PROCEDURE

Equations (10a) and (10b) represent the final forms of the momentum equation to be used in solving for the Mach number at either station x or $x+\Delta x$. If the conditions are known at station x, the Mach number at station $x+\Delta x$ is determined from equation (10a); while if the conditions are known at station $x+\Delta x$, the Mach number at station x is determined from equation (10b). Increasing values of x, corresponding to a positive Δx, must be in the flow direction. A given duct can be divided into an arbitrary number of increments in Δx (not necessarily of the same length), and the Mach number change through that duct can then be computed by progressing stepwise from the end where all conditions are known. The size of each increment should depend upon the accuracy desired. Subsequent examples will indicate the convergence of the solution as Δx is decreased.

Both $F(M,\mathbf{A})$ and $G(M)$ have been plotted in figure 2 for air with $\gamma = 1.40$ to expedite the solution. Since γ has a secondary effect on the Mach number change and the variation of γ encountered in air flow is small, the consideration of other values of γ is unwarranted.

The procedure for determining the Mach number change and thus the pressure change with air flowing in a duct is to use equation (10a) or (10b) and figures 1 and 2. For example, the procedure for determining the Mach number at station x from the conditions at station $x+\Delta x$ is to first evaluate $\mathbf{A}_{x+\Delta x}$ (eq. (14)). From the value of the Mach number at station $x+\Delta x$ and $\mathbf{A}_{x+\Delta x}$, the values of $F(M_{x+\Delta x},\mathbf{A}_{x+\Delta x})$ and $G(M_{x+\Delta x})$ are read from figure 2. Then, with the specified values of $\omega^2 r_{x+\Delta x}\Delta x/T'_{x+\Delta x}$ and $\sqrt{T'_{x+\Delta x}/T'_x}\ (A_x/A_{x+\Delta x})$, $F(M_x,\mathbf{A}_x)$ is calculated from equation (10b). From $F(M_x,\mathbf{A}_x)$ and $\mathbf{A}_x$ (eq. (13)), the Mach number at station x is read from figure 2. Once the Mach numbers are known, the pressure change can be determined from figure 1. To illustrate this procedure, a typical sample calculation for an air-cooled turbine blade including heat transfer, friction, rotation, and area change has been carried out in table I with the passage divided into two equal parts. Double columns (column 9 and columns 20 to 27) are included in the sample calculation setup in table I so that the calculations could proceed from either the x or $x+\Delta x$ station. For each of these double columns, the left column is applicable if the solution

is initiated at station $x+\Delta x$, while the right column is applicable if the solution is initiated at station x. The first 11 columns of table I give the conditions which must be prescribed. Although the friction coefficient f is assumed to be the same at both the x and $x+\Delta x$ stations, it may be allowed to vary if greater accuracy seems warranted. The remaining columns of table I give the computations required to determine the Mach number at either station x or $x+\Delta x$. The sample calculation shown is for the determination of the Mach number at station x from conditions at station $x+\Delta x$.

RESULTS AND DISCUSSION

In order to evaluate the accuracy of the solution presented herein, calculations were made for conditions typical of those occurring for air-cooled turbine blades (1) by use of the charts and (2) by the method of numerical integration presented in reference 3. The results of these calculations employing equation (10b) with 1, 2, and 4 steps with increments of equal length are tabulated in table II and compared with the results obtained from the more detailed numerical methods of reference 3. Three cases are illustrated in table II: (I) heat transfer and friction; (II) heat transfer, friction, and rotation; (III) heat transfer, friction, rotation, and area change. For the first two cases, the air temperature was assumed to vary linearly from the inlet to the exit. However, for the third case, the air temperature was varied exponentially to correspond with flow inside a duct heated by crossflow and with constant inside and outside surface heat-transfer coefficients. The total pressures used for evaluating the percentage differences between those obtained by the two solutions were computed from figure 1 and the known conditions and areas.

It is shown in table II that the solutions converge rapidly. In addition, the comparison between the two solutions shows that good agreement is obtained by using only two steps. Even for the third case, the computed total pressures agreed within 4 percent for only two steps. In this case the disagreement was essentially eliminated by using four steps. Solutions were also made with eight steps, but for the conditions used in the examples of table II, the improvement over the use of four steps was probably not within the accuracy of the charts. The approximations given by equations (6) to (8) are least accurate at the higher flow Mach numbers where slight changes in the total pressure cause large changes in the Mach number (fig. 1) and, therefore, large changes in the velocity and state conditions. For this reason, the Mach numbers used in the examples of table II are intentionally high. It should be pointed out that the accuracy of the approximations employed in equations (10) are dependent upon Mach number changes, and not the passage length, when rotational forces are neglected. Thus, the accuracies shown by the results of table II are indicative of the accuracies which would be obtained for longer passages with similar Mach number changes due to heat transfer, friction, and area change alone.

In many cases the air temperature varies exponentially with x as in case III of table II. In order to check the effect on the Mach number change of using the more easily obtainable linear air temperature distribution instead of an exponential variation, the examples presented in table II were recomputed as follows: (1) Cases I and II, which were calculated for table II with a linear air temperature distribution, were determined for an exponential distribution, and (2) case III was computed for a linear variation instead of the exponential air temperature distribution used for table II. The results of these calculations showed that the differences between the two variations were negligible and probably of the same order of magnitude as the accuracy of the charts. For this reason, it appears that although the air temperature might vary exponentially with x, for convenience a linear variation may be assumed for evaluating the temperatures needed for the incremental subdivisions.

CONCLUDING REMARKS

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A method is developed for the rapid determination of the Mach number change and pressure change for the flow of a compressible fluid in a duct with heat transfer, friction, rotation, and area change. The method uses a step-by-step integration which, when applied to specific examples typical for air-cooled turbine blades, converged rapidly and provided good accuracy with, at most, only four steps. In addition, for these specific examples, a negligible difference occurred between using a linear or an exponential temperature variation along the duct length. This result indicates that the more convenient linear temperature variation may be used for evaluating the temperatures for the incremental subdivisions if the actual temperature varies exponentially, as frequently occurs in practice.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
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APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	flow area, sq ft
b	duct length or span, ft
D_h	hydraulic diameter, $4A/l$, ft
F_b	generalized body force, lb/lb of coolant, $\omega^2 r/g$ (eq. (3))
F_d	drag force, lb
$F(M,A)$	$\dfrac{1}{M\left(1+\frac{\gamma-1}{2}M^2\right)^{1/2}}\left[1-\dfrac{\gamma M^2}{4}(A-4)\right]$ (eq. (11))
f	friction coefficient
$G(M)$	$\dfrac{\left(1+\frac{\gamma-1}{2}M^2\right)^{1/2}}{gRM}$ (eq. (12))
g	standard acceleration due to gravity, 32.2 ft/sec^2
l	wetted perimeter, ft
M	Mach number relative to passage
P	static pressure, lb/sq ft abs
P'	total pressure with respect to passage, lb/sq ft
R	gas constant, 53.3 ft-lb/(lb)($^{\circ}$R)
r	radius, ft
S	surface area, sq ft
T	static temperature, $^{\circ}$R
T'	total temperature with respect to passage, $^{\circ}$R

V	velocity relative to passage, ft/sec
w	weight flow, lb/sec
x	distance measured from passage inlet in flow direction, ft
γ	ratio of specific heats, 1.40
Λ_x	$\frac{4f_x\Delta x}{D_{h,x}} - 2\left(\frac{A_x}{A_{x+\Delta x}} - 1\right)$ (eq. (13))
$\Lambda_{x+\Delta x}$	$-\frac{4f_{x+\Delta x}\Delta x}{D_{h,x+\Delta x}} - 2\left(\frac{A_{x+\Delta x}}{A_x} - 1\right)$ (eq. (14))
ρ	mass density, slugs/cu ft
Ω	$\frac{\omega^2 r\Delta x}{T'}$
ω	angular velocity, radians/sec

Subscripts:

e	exit of passage
i	inlet of passage
x	distance measured from passage inlet in flow direction, ft

APPENDIX B

DERIVATION OF APPROXIMATE MOMENTUM EQUATION

Upon collecting terms and employing the relations

$$(\rho V)_{x+\Delta x}\left(\frac{V_x}{2}\right) = (\rho V)_{x+\Delta x}\frac{(\rho V)_x}{(\rho V)_x}\left(\frac{V_x}{2}\right) = \frac{(\rho V^2)_x}{2}\frac{(\rho V)_{x+\Delta x}}{(\rho V)_x}$$

and

$$(\rho V)_x\left(\frac{V_{x+\Delta x}}{2}\right) = (\rho V)_x\frac{(\rho V)_{x+\Delta x}}{(\rho V)_{x+\Delta x}}\left(\frac{V_{x+\Delta x}}{2}\right) = \frac{(\rho V^2)_{x+\Delta x}}{2}\frac{(\rho V)_x}{(\rho V)_{x+\Delta x}}$$

equation (9a) becomes

$$P_{x+\Delta x} - P_x - \frac{(\rho V^2)_{x+\Delta x}}{4}\left[-2 - 2\frac{(\rho V)_x}{(\rho V)_{x+\Delta x}} - \frac{4f_{x+\Delta x}\Delta x}{D_{h,x+\Delta x}}\right] +$$
$$\frac{(\rho V^2)_x}{4}\left[-2 - 2\frac{(\rho V)_{x+\Delta x}}{(\rho V)_x} + \frac{4f_x\Delta x}{D_{h,x}}\right] - \omega^2 r_x\rho_x\Delta x = 0 \tag{20a}$$

and equation (9b) becomes

$$P_{x+\Delta x} - P_x - \frac{(\rho V^2)_{x+\Delta x}}{4}\left[-2 - 2\frac{(\rho V)_x}{(\rho V)_{x+\Delta x}} - \frac{4f_{x+\Delta x}\Delta x}{D_{h,x+\Delta x}}\right] +$$
$$\frac{(\rho V^2)_x}{4}\left[-2 - 2\frac{(\rho V)_{x+\Delta x}}{(\rho V)_x} + \frac{4f_x\Delta x}{D_{h,x}}\right] - \omega^2 r_{x+\Delta x}\rho_{x+\Delta x}\Delta x = 0 \tag{20b}$$

If the continuity equation is employed to obtain the relation

$$w = (\rho VA)_x = (\rho VA)_{x+\Delta x}$$

or

$$\frac{(\rho V)_x}{(\rho V)_{x+\Delta x}} = \frac{A_{x+\Delta x}}{A_x} \tag{21}$$

and if the simplifying notations

$$\Lambda_x = \frac{4f_x\Delta x}{D_{h,x}} - 2\left(\frac{A_x}{A_{x+\Delta x}} - 1\right) \tag{13}$$

$$\Lambda_{x+\Delta x} = -\frac{4f_{x+\Delta x}\Delta x}{D_{h,x+\Delta x}} - 2\left(\frac{A_{x+\Delta x}}{A_x} - 1\right) \tag{14}$$

$$\Omega = \frac{\omega^2 r\Delta x}{T'}$$

are used, equation (20a) becomes

$$P_{x+\Delta x} - P_x - \frac{(\rho V^2)_{x+\Delta x}}{4}(\Lambda_{x+\Delta x} - 4) + \frac{(\rho V^2)_x}{4}(\Lambda_x - 4) - \Omega_x T_x' \rho_x = 0 \tag{22a}$$

and equation (20b) becomes

$$P_{x+\Delta x} - P_x - \frac{(\rho V^2)_{x+\Delta x}}{4}(\Lambda_{x+\Delta x} - 4) + \frac{(\rho V^2)_x}{4}(\Lambda_x - 4) - \Omega_{x+\Delta x} T_{x+\Delta x}' \rho_{x+\Delta x} = 0 \tag{22b}$$

A more convenient form of equations (22a) and (22b) can be obtained if the variables are changed by employing the equation of state, the energy equation, and the continuity equation. From the equation of state, ρV^2 can be expressed as

$$\rho V^2 = \frac{P}{gRT} V^2 = \gamma P M^2 \tag{23}$$

In addition, the static pressure can be eliminated by using the continuity equation and the equation of state as follows:

$$w = g\rho AV = \frac{P}{RT} AV \tag{24}$$

or

$$P = \frac{w}{A}\frac{RT}{V} = \frac{w}{A}\sqrt{\frac{T'R}{\gamma g}}\sqrt{\frac{T}{T'}}\frac{1}{M} \tag{25}$$

where the total- to static-temperature ratio can be eliminated by using the energy equation in the form

$$\frac{T'}{T} = 1 + \frac{\gamma-1}{2} M^2 \tag{17}$$

to give the expression

$$P = \frac{w}{A}\sqrt{\frac{T'R}{\gamma g}} \frac{1}{M\left(1 + \frac{\gamma-1}{2} M^2\right)^{1/2}} \tag{26}$$

Finally, application of the continuity equation gives

$$T'\rho = \frac{wT'\rho}{g\rho AV} = \frac{wT'}{gA} \frac{1}{V} \tag{27}$$

$$T'\rho = \frac{w}{Ag} \sqrt{\frac{T'}{\gamma gR}} \sqrt{\frac{T'}{T}} \frac{1}{M} \tag{28}$$

or, by incorporating equation (17) in equation (28),

$$T'\rho = \frac{w}{Ag} \sqrt{\frac{T'}{\gamma gR}} \frac{\left(1 + \frac{\gamma-1}{2} M^2\right)^{1/2}}{M} \tag{29}$$

If equations (23), (26), and (29) are substituted into equations (22a) and (22b) and $w\sqrt{R/\gamma g}$ is factored out, the two forms of the approximate momentum equation become

$$\frac{\sqrt{T'_{x+\Delta x}}}{A_{x+\Delta x}} \frac{1}{M_{x+\Delta x}\left(1 + \frac{\gamma-1}{2} M^2_{x+\Delta x}\right)^{1/2}}\left[1 - \frac{\gamma M^2_{x+\Delta x}}{4} (\Lambda_{x+\Delta x} - 4)\right] -$$

$$\frac{\sqrt{T'_x}}{A_x} \frac{1}{M_x\left(1 + \frac{\gamma-1}{2} M_x^2\right)^{1/2}}\left[1 - \frac{\gamma M_x^2}{4} (\Lambda_x - 4)\right] - \Omega_x \frac{\sqrt{T'_x}}{A_x gR} \frac{\left(1 + \frac{\gamma-1}{2} M_x^2\right)^{1/2}}{M_x} = 0$$

(30a)

and

$$\frac{\sqrt{T'_{x+\Delta x}}}{A_{x+\Delta x}} \frac{1}{M_{x+\Delta x}\left(1 + \frac{\gamma-1}{2} M^2_{x+\Delta x}\right)^{1/2}}\left[1 - \frac{\gamma M^2_{x+\Delta x}}{4} (\Lambda_{x+\Delta x} - 4)\right] -$$

$$\frac{\sqrt{T'_x}}{A_x} \frac{1}{M_x\left(1 + \frac{\gamma-1}{2} M^2_x\right)^{1/2}}\left[1 - \frac{\gamma M^2_x}{4} (\Lambda_x - 4)\right] - \Omega_{x+\Delta x} \frac{\sqrt{T'_{x+\Delta x}}}{A_{x+\Delta x} gR} \frac{\left(1 + \frac{\gamma-1}{2} M^2_{x+\Delta x}\right)^{1/2}}{M_{x+\Delta x}} = 0 \tag{30b}$$

Upon simplifying, rearranging, and employing the definition of Ω, equation (30a) becomes

$$F(M_{x+\Delta x}, \Lambda_{x+\Delta x}) = \sqrt{\frac{T'_x}{T'_{x+\Delta x}}} \left(\frac{A_{x+\Delta x}}{A_x}\right)\left[F(M_x, \Lambda_x) + \frac{\omega^2 r_x \Delta x}{T'_x} G(M_x)\right] \tag{10a}$$

and equation (30b) becomes

$$F(M_x, \Lambda_x) = \sqrt{\frac{T'_{x+\Delta x}}{T'_x}} \left(\frac{A_x}{A_{x+\Delta x}}\right)\left[F(M_{x+\Delta x}, \Lambda_{x+\Delta x}) - \frac{\omega^2 r_{x+\Delta x} \Delta x}{T'_{x+\Delta x}} G(M_{x+\Delta x})\right] \tag{10b}$$

where

$$F(M, \Lambda) = \frac{1}{M\left(1 + \frac{\gamma-1}{2} M^2\right)^{1/2}}\left[1 - \frac{\gamma M^2}{4} (\Lambda - 4)\right] \tag{11}$$

$$G(M) = \frac{\left(1 + \frac{\gamma-1}{2} M^2\right)^{1/2}}{gRM} \tag{12}$$

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TABLE I. - SAMPLE CALCULATION AND SETUP

Step	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)		(10)	(11)	(12)	(13)	(14)	(15)
	b, ft	Δx, ft	A_x, sq ft	$A_{x+\Delta x}$, sq ft	$D_{h,x}$, ft	$D_{h,x+\Delta x}$, ft	T'_x, °R	$T'_{x+\Delta x}$, °R	$r_{x+\Delta x}$, ft	r_x, ft	ω, rad/sec	f	$A_x/A_{x+\Delta x}$, (3)/(4)	$\frac{A_{x+\Delta x}}{A_x}$, 1/(12)	$\frac{T'_x}{T'_{x+\Delta x}}$, (7)/(8)	$\frac{4f\Delta x}{D_{h,x}}$, $\frac{4(11)(2)}{(5)}$
1	0.3	0.15	0.000409	0.000194	0.00668	0.0044	970	1160	1.417		796	0.0065	2.108	0.4744	0.8362	0.5838
2	0.3	0.15	0.000590	0.000409	0.00749	0.00668	763	970	1.267		796	0.0065	1.443	0.6930	0.7866	0.5207

Step	(16)	(17)	(18)	(19)	(20)		(21)		(22)	
	$\frac{4f\Delta x}{D_{h,x+\Delta x}}$, $\frac{4(11)(2)}{(6)}$	A_x, (15)-2[(12)-1]	$A_{x+\Delta x}$, -(16)-2[(13)-1]	$\frac{A_{x+\Delta x}}{A_x}\sqrt{\frac{T'_x}{T'_{x+\Delta x}}}$, $(13)(14)^{1/2}$	$\frac{\omega^2 r_{x+\Delta x}\Delta x}{T'_{x+\Delta x}}$, $\frac{(10)^2(9)(2)}{(8)}$	$\frac{\omega^2 r_x\Delta x}{T'_x}$, $\frac{(10)^2(9)(2)}{(7)}$	$M_{x+\Delta x}$	M_x	$G(M_{x+\Delta x})$ $\times 10^3$	$G(M_x)$ $\times 10^3$
1	0.8864	-1.632	0.1648	0.4337	116.1		0.629		0.963	
2	0.5838	-0.3653	0.0302	0.6146	124.2		0.2105		2.80	

Step	(23)		(24)		(25)		(26)		(27)	
	$F(M_{x+\Delta x},A_{x+\Delta x})$	$F(M_x,A_x)$	$\frac{\omega^2 r_{x+\Delta x}\Delta x}{T'_{x+\Delta x}}\times(G(M_{x+\Delta x}))$, (20)(22)	$\frac{\omega^2 r_x\Delta x}{T'_x}\times(G(M_x))$, (20)(22)	$\left[F(M_{x+\Delta x},A_{x+\Delta x}) - \frac{\omega^2 r_{x+\Delta x}\Delta x}{T'_{x+\Delta x}}G(M_{x+\Delta x})\right]$, (23)-(24)	$\left[F(M_x,A_x) + \frac{\omega^2 r_x\Delta x}{T'_x}G(M_x)\right]$, (23)+(24)	$F(M_x,A_x)$, $\left[\frac{(25)}{(19)}\right]_x$	$F(M_{x+\Delta x},A_{x+\Delta x})$, $[(25)(19)]_{x+\Delta x}$	M_x	$M_{x+\Delta x}$
1	2.345		0.1118		2.233		5.148		0.2105	
2	5.027		0.348		4.679		7.613		0.1350	

TABLE II. - COMPARISON OF RESULTS FROM NUMERICAL AND GRAPHICAL SOLUTIONS

Case	$\frac{T'_i}{T'_e}$	$\frac{A_i}{A_e}$	Exit Mach number, M_e	Inlet Mach number M_i from ref. 3	Inlet Mach number M_i from charts			$\frac{(P'_i)_{ref} - (P'_i)_{chart}}{(P'_i)_{ref}}$		
					Trial			Trial		
					1	2	4	1	2	4
[a]I	0.7595	1.0	0.833	0.419	0.406	0.416	0.418	0.0310	0.007	0.003
[b]II	.6864	1.0	.800	.505	.472	.498	.501	.0510	.009	.006
[c]III	.658	3.04	.629	.141	.1232	.135	.140	.134	.042	.009

[a]Heat transfer and friction.

[b]Heat transfer, friction, and rotation.

[c]Heat transfer, friction, rotation, and area change.

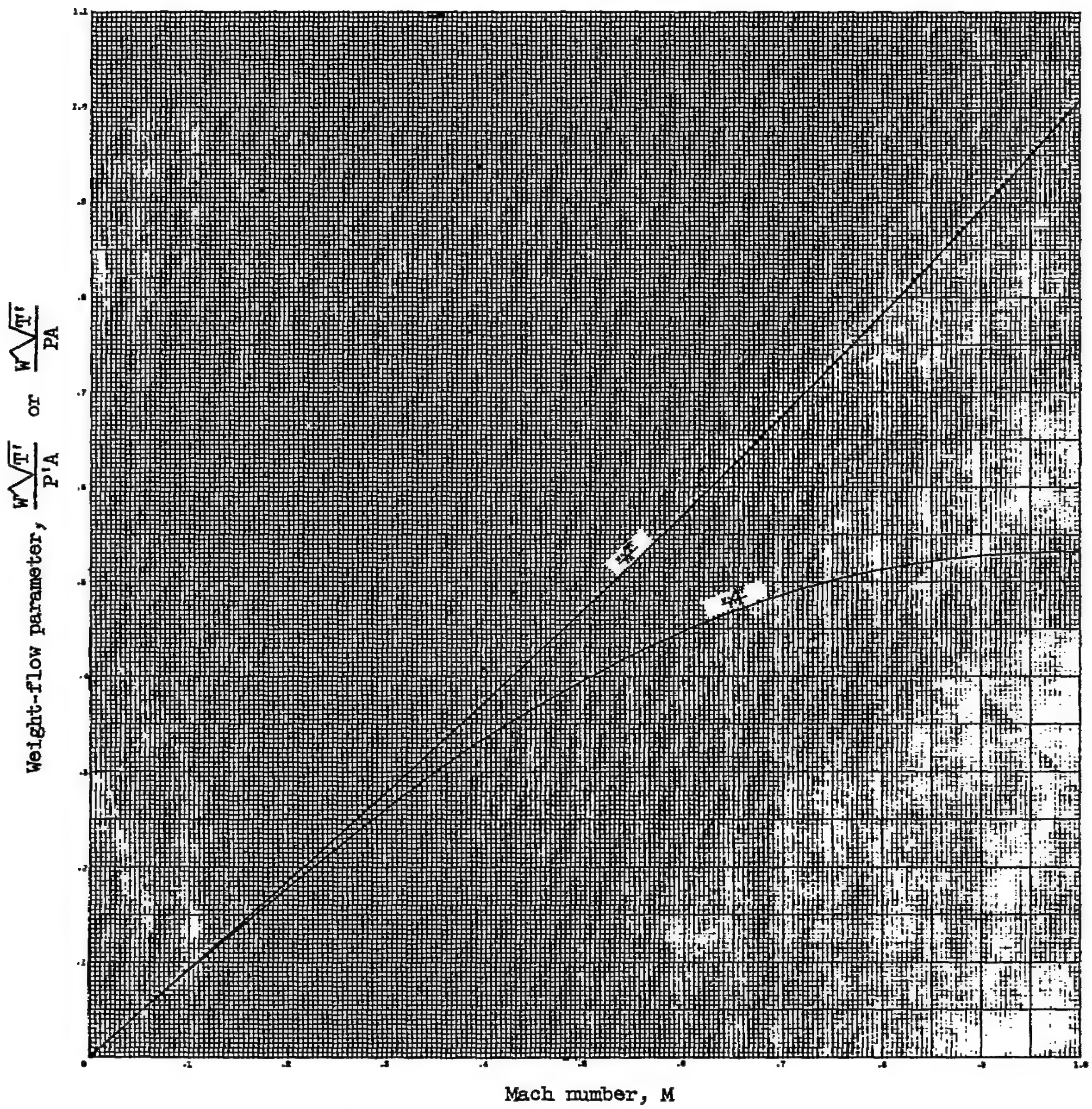
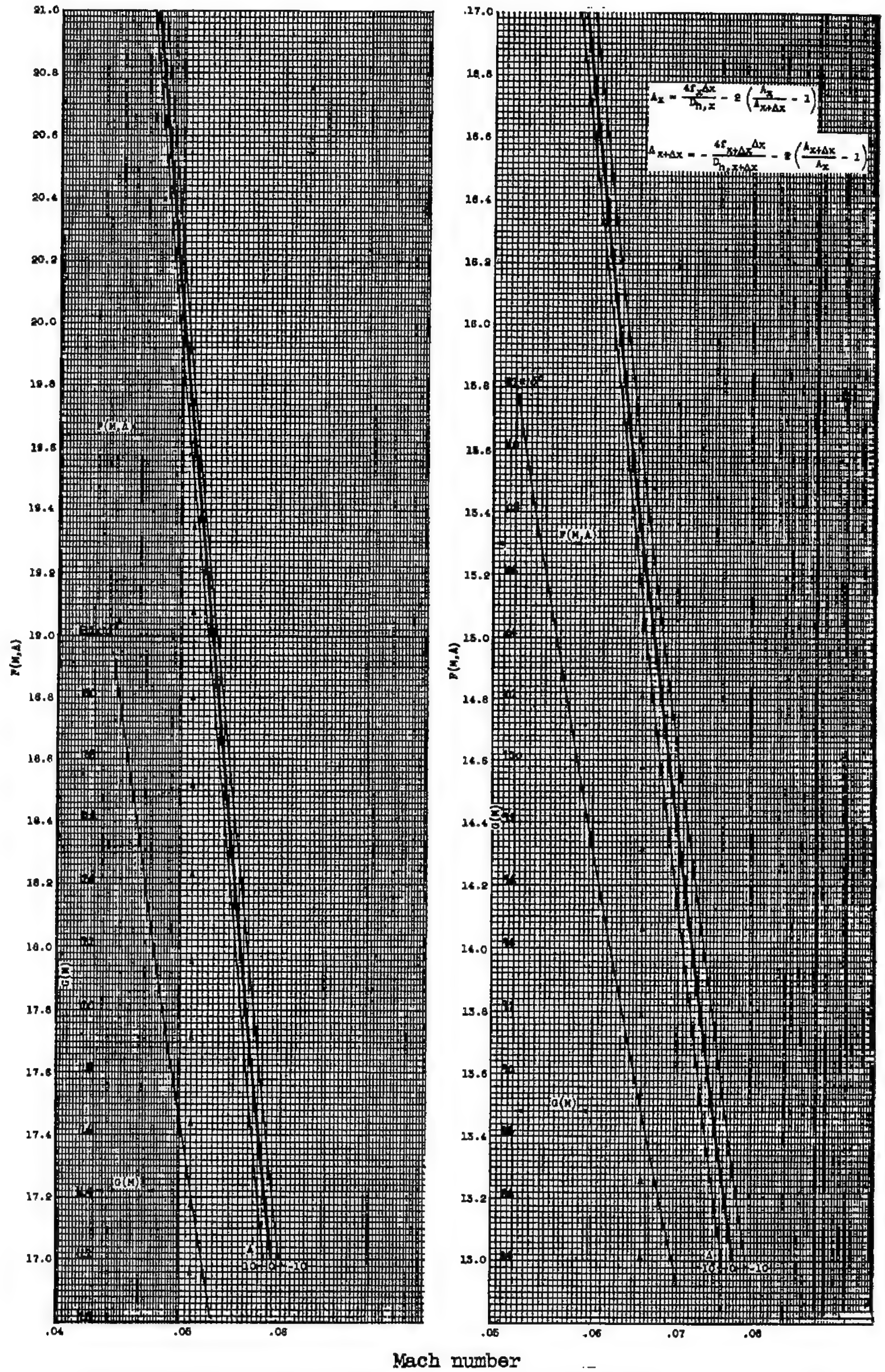
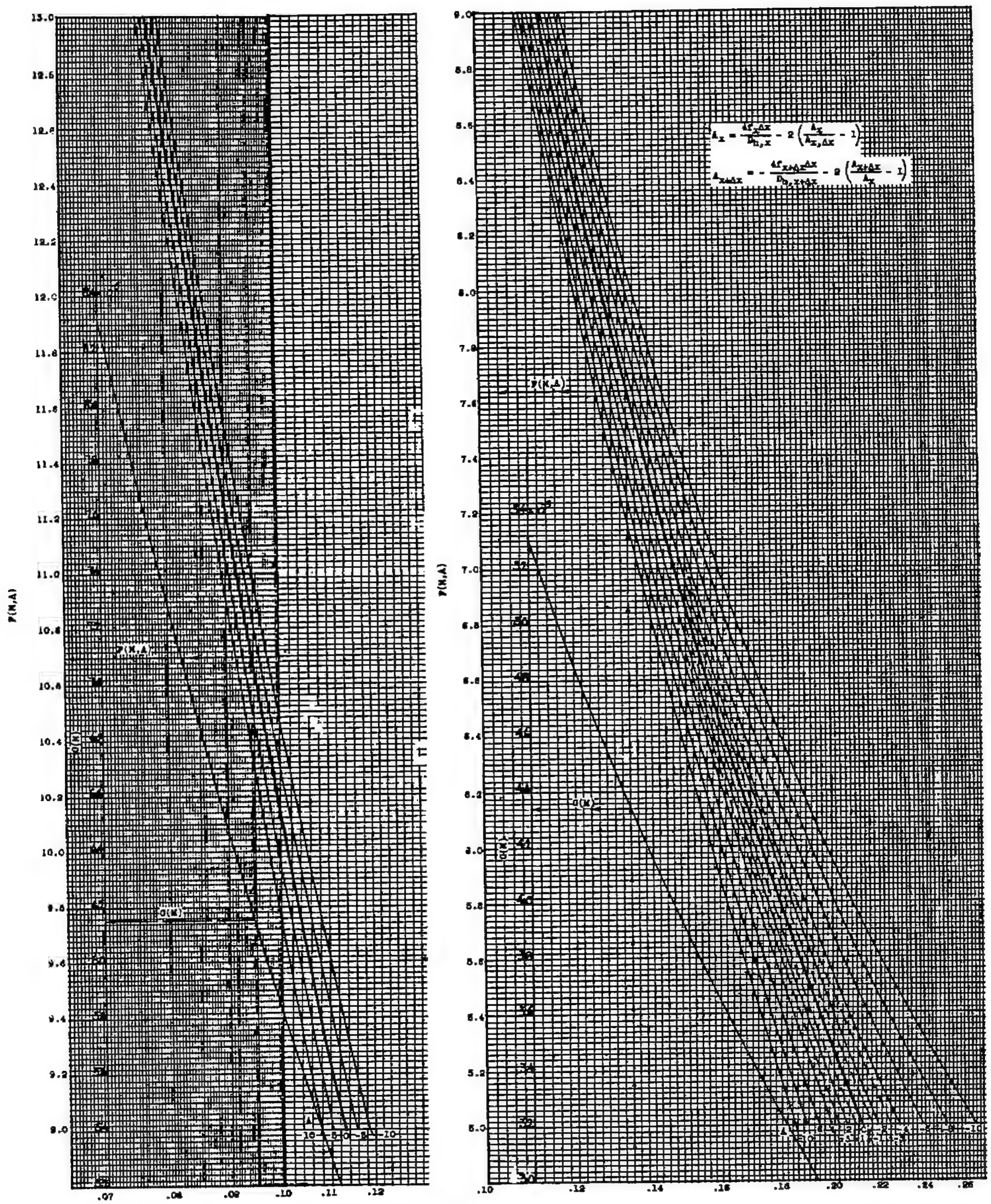


Figure 1. - Evaluation of weight-flow parameter from Mach number (ratio of specific heats, 1.40). (A $15\frac{1}{2}$- by 17-in. print of this fig. is attached.)



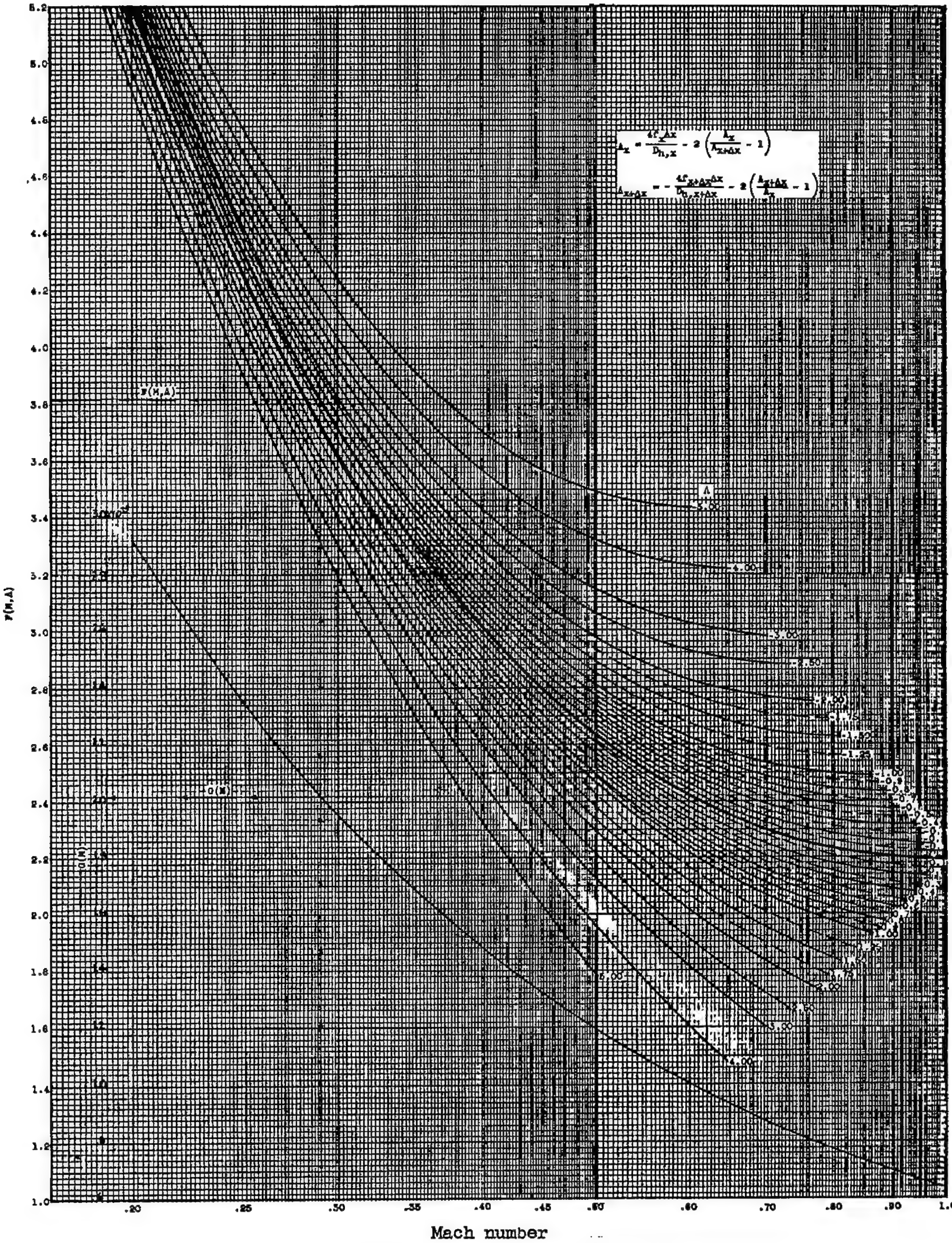
(a) Mach number range, 0.045 to 0.080.

Figure 2. - Evaluation of F(M,Λ) and G(M) (ratio of specific heats, 1.40). (A 12- by 20-in. print of this fig. is attached.)



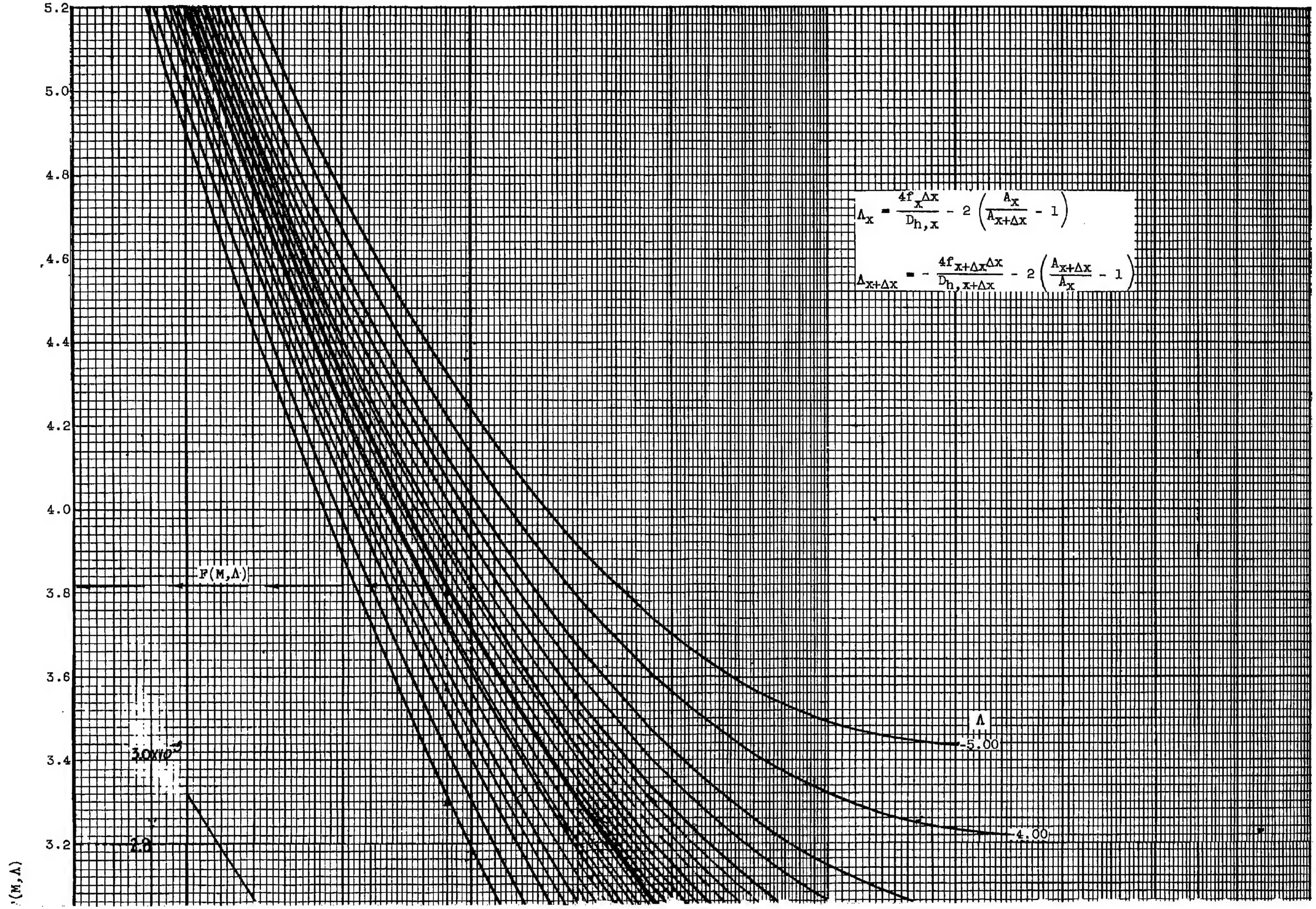
(b) Mach number range, 0.075 to 0.25.

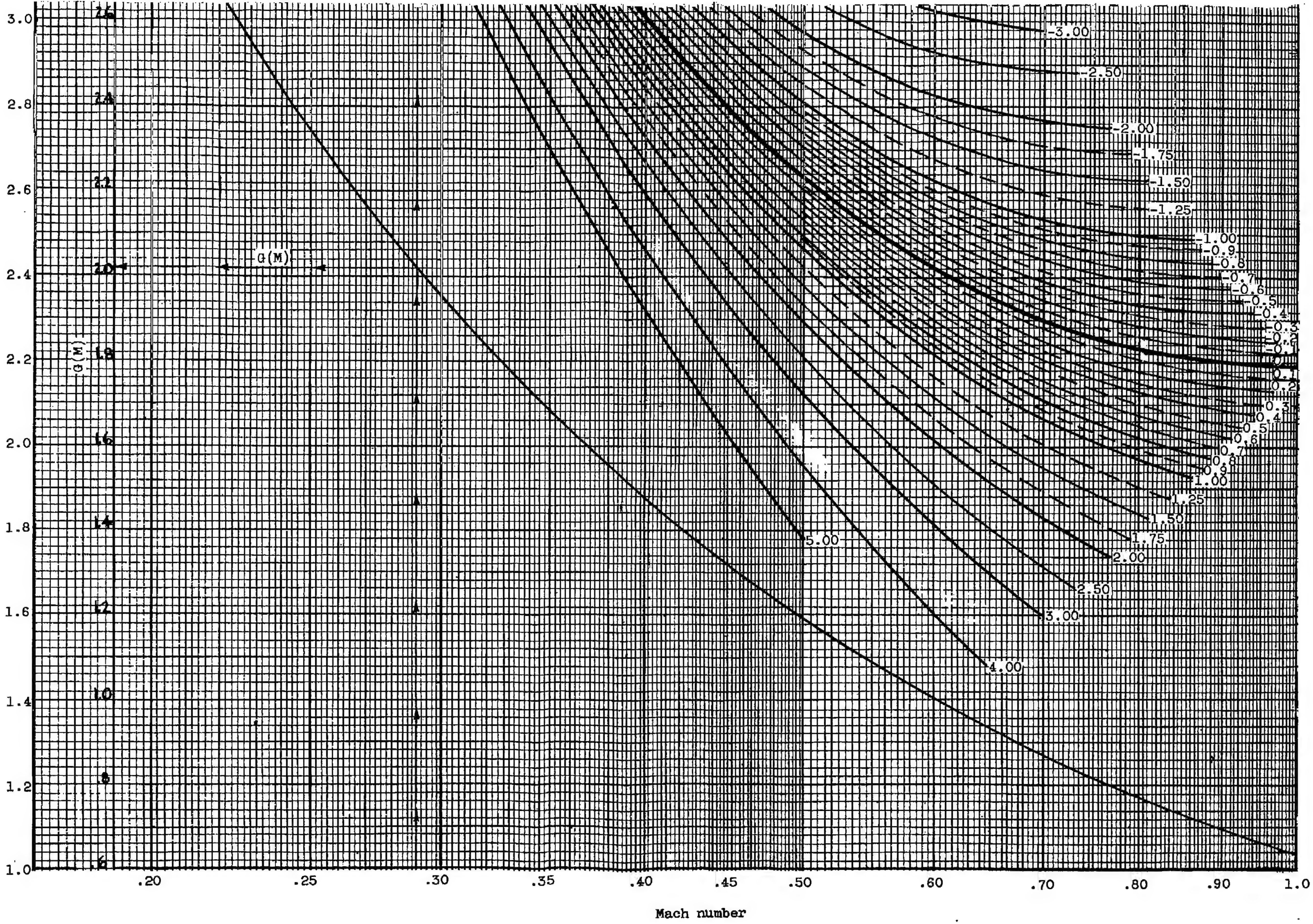
Figure 2. - Continued. Evaluation of F(M,Λ) and G(M) (ratio of specific heats, 1.40). (A $14\frac{1}{2}$- by 20-in. print of this fig. is attached.)



(c) Mach number range, 0.20 to 1.0.

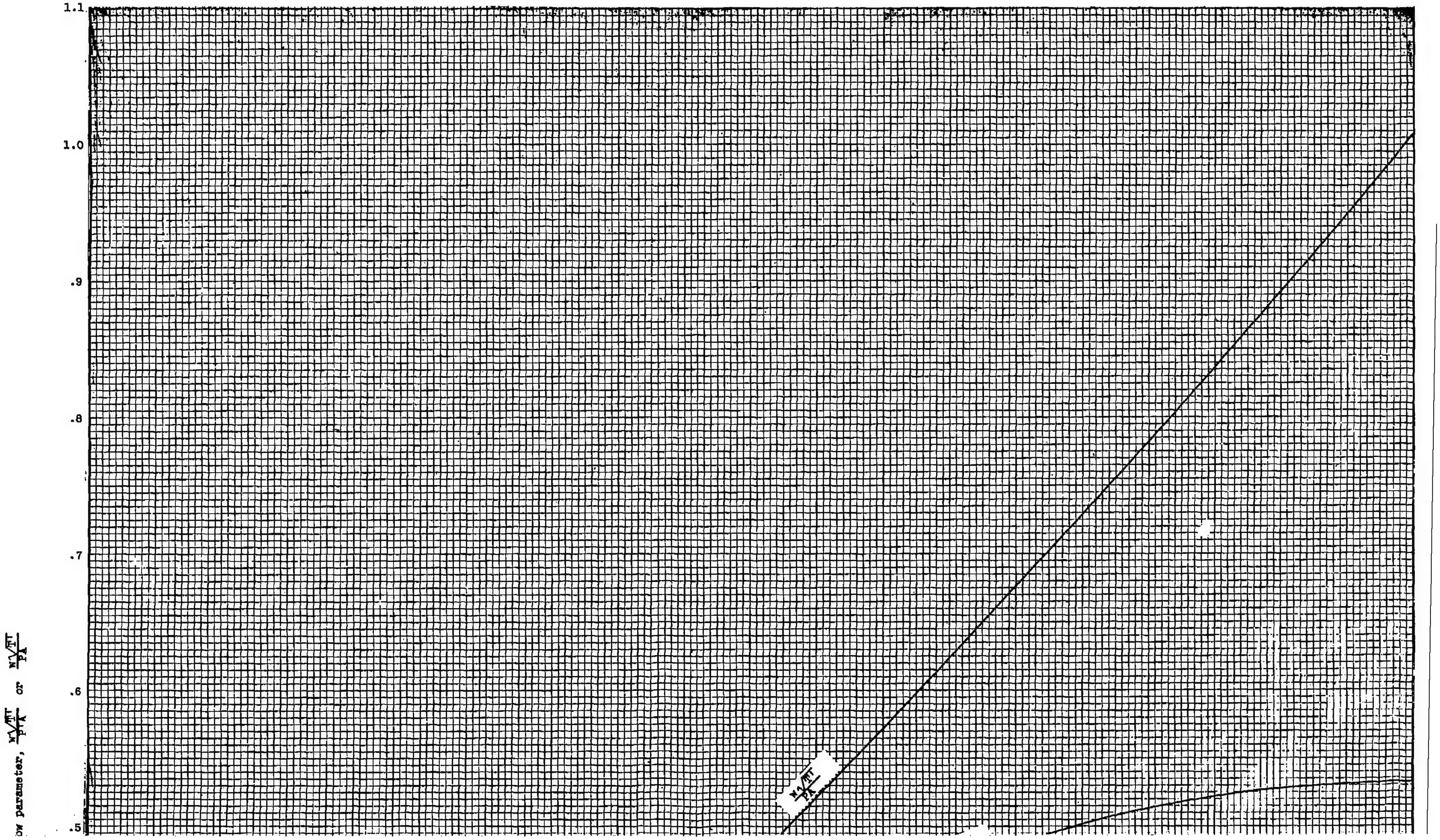
Figure 2. - Concluded. Evaluation of $F(M,\Lambda)$ and $G(M)$ (ratio of specific heats, 1.40). (A $14\frac{1}{2}$- by 20-in. print of this fig. is attached.)


5.2
5.0
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
3.2
F(M,Λ)
3.0x10^-3
2.8
Λ
-5.00
-4.00
Λx = 4fxΔx/Dh,x - 2(Ax/Ax+Δx - 1)
Λx+Δx = - 4fx+ΔxΔx/Dh,x+Δx - 2(Ax+Δx/Ax - 1)
(M,Λ)



(c) Mach number range, 0.20 to 1.0.

Figure 2. - Concluded. Evaluation of $F(M,\Lambda)$ and $G(M)$ (ratio of specific heats, 1.40).


1.1
1.0
.9
.8
.7
.6
.5
ow parameter, $\frac{W\sqrt{T}}{PA}$ or $\frac{W\sqrt{T'}}{P'A}$
$\frac{W\sqrt{T'}}{P'A}$

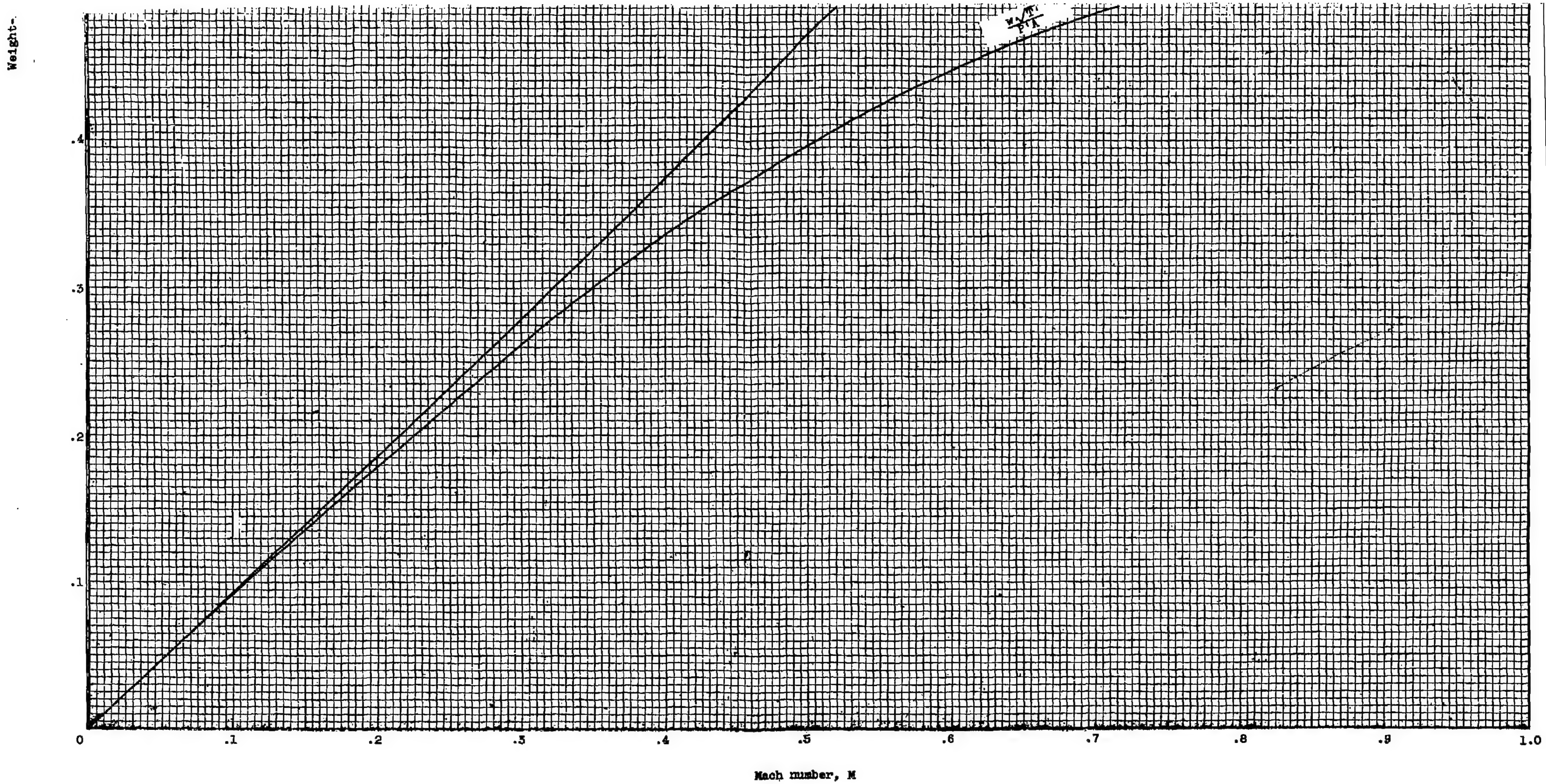
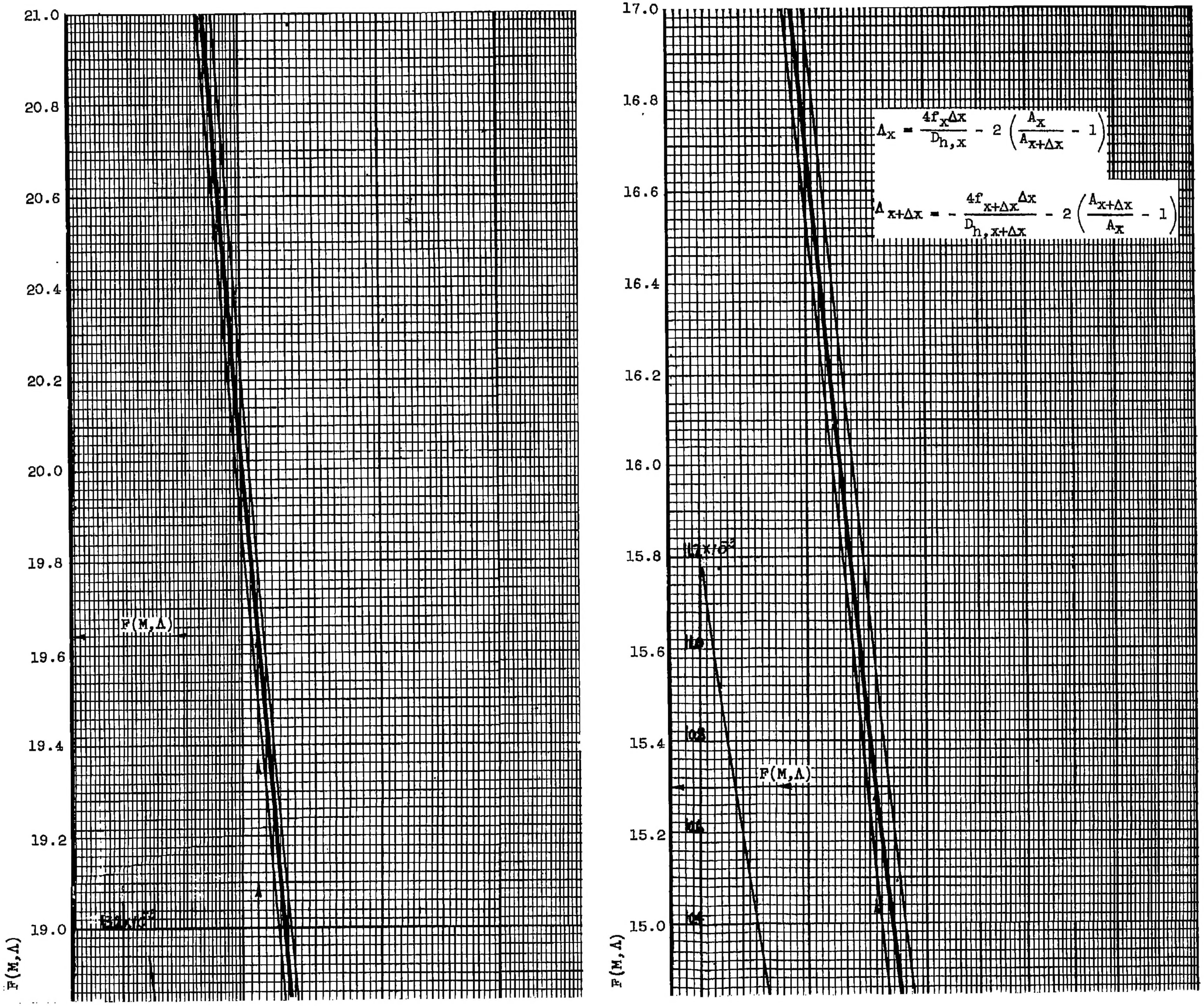
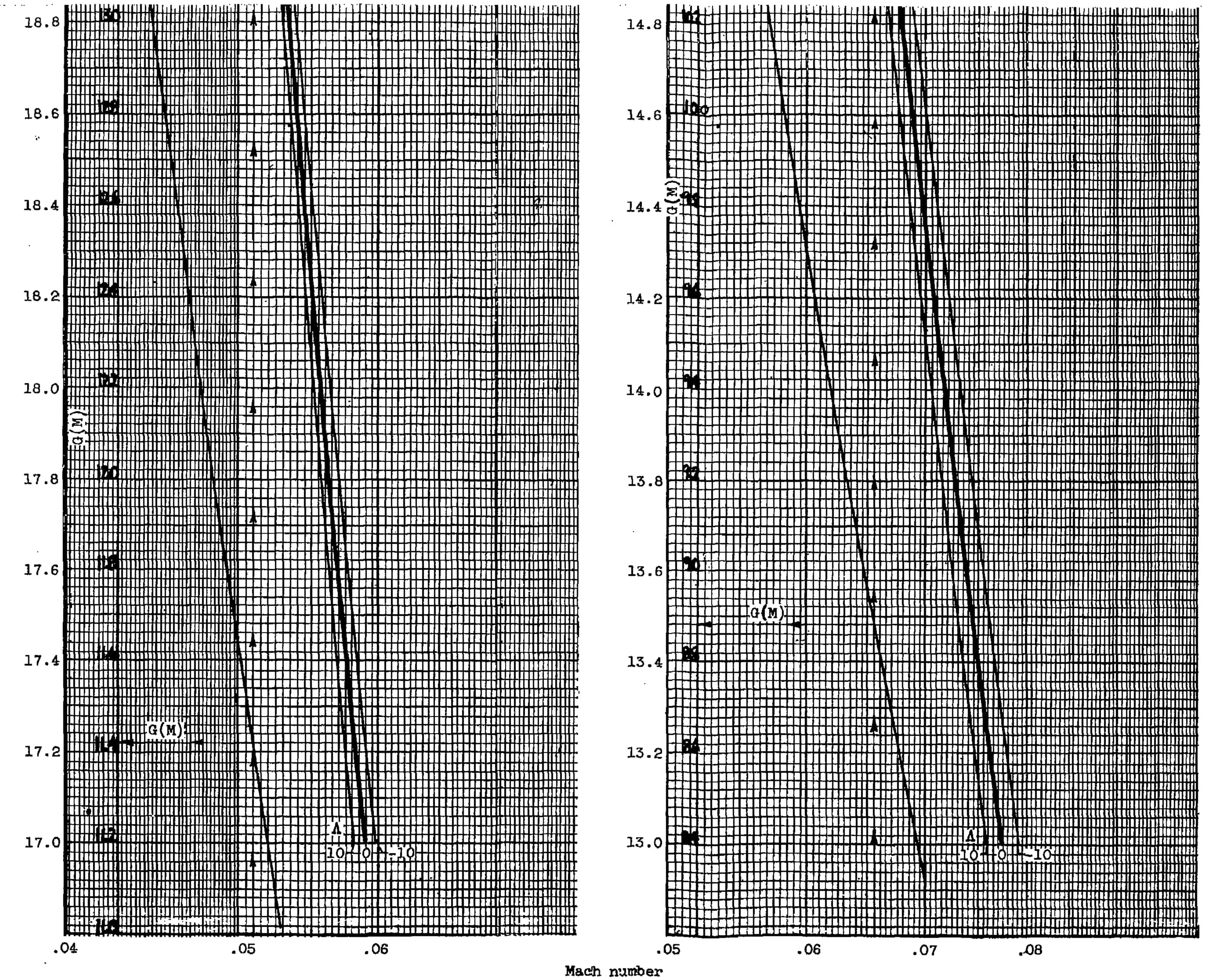


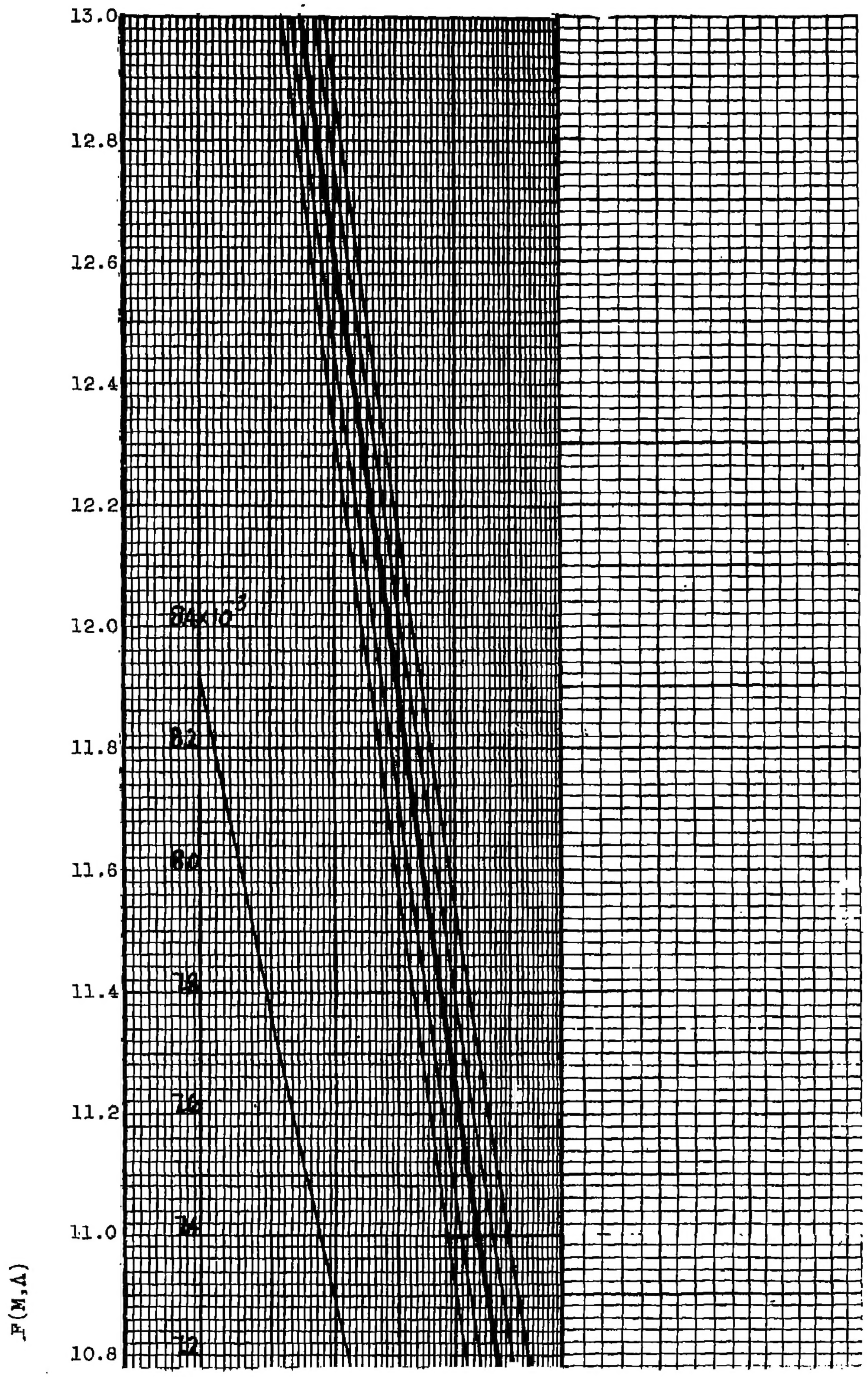
Figure 1. - Evaluation of weight-flow parameter from Mach number (ratio of specific heats, 1.40).

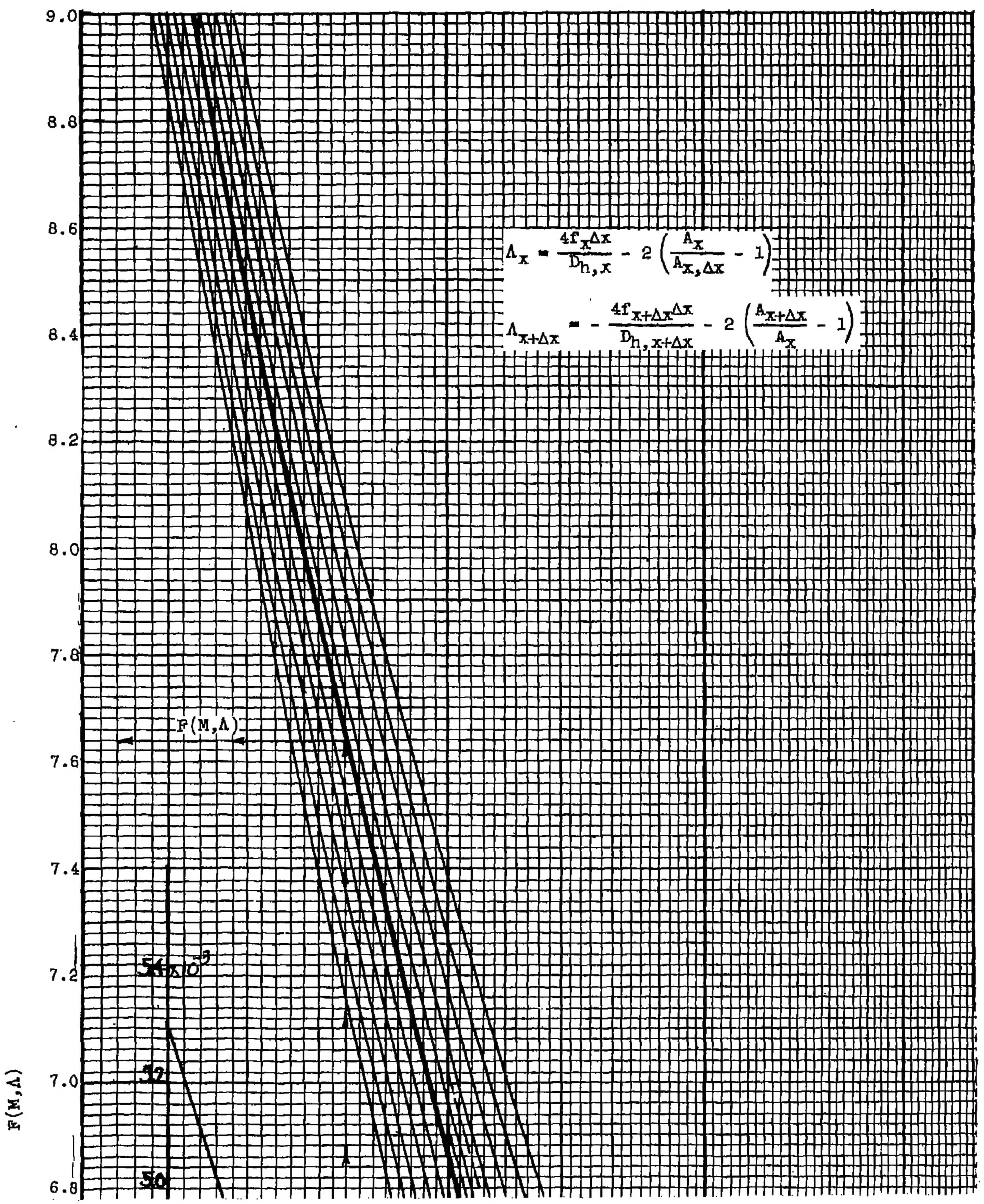

21.0
20.8
20.6
20.4
20.2
20.0
19.8
19.6
19.4
19.2
19.0
F(M,Λ)
F(M,Λ)
17.0
16.8
16.6
16.4
16.2
16.0
15.8
15.6
15.4
15.2
15.0
F(M,Λ)
F(M,Λ)
$\Lambda_x = \frac{4f_x\Delta x}{D_{h,x}} - 2\left(\frac{A_x}{A_{x+\Delta x}} - 1\right)$
$\Lambda_{x+\Delta x} = -\frac{4f_{x+\Delta x}\Delta x}{D_{h,x+\Delta x}} - 2\left(\frac{A_{x+\Delta x}}{A_x} - 1\right)$



(a) Mach number range, 0.045 to 0.080.

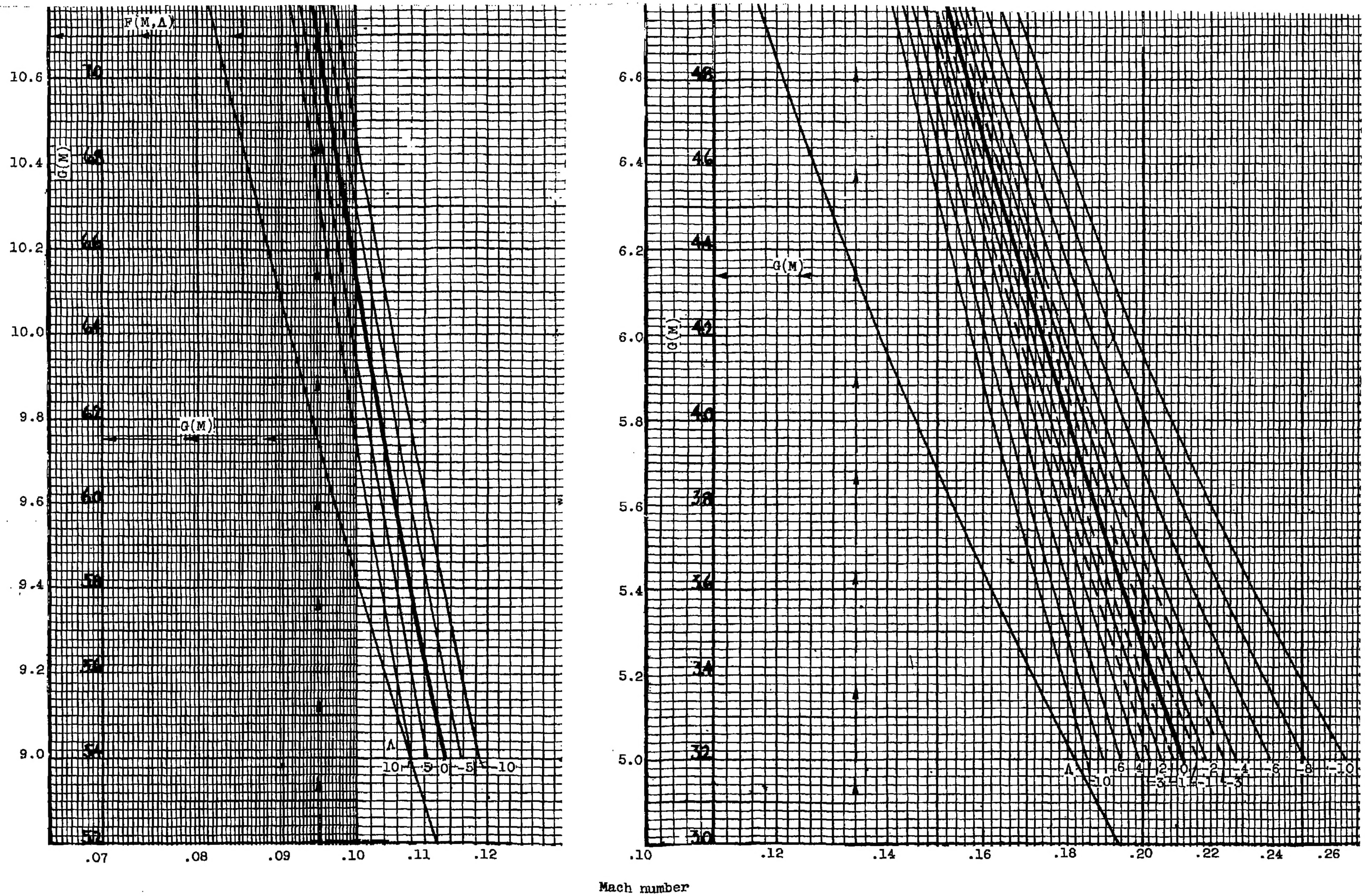
Figure 2. - Evaluation of $F(M,\Lambda)$ and $G(M)$ (ratio of specific heats, 1.40).





$$\Lambda_x = \frac{4f_x \Delta x}{D_{h,x}} - 2\left(\frac{A_x}{A_{x,\Delta x}} - 1\right)$$

$$\Lambda_{x+\Delta x} = -\frac{4f_{x+\Delta x}\Delta x}{D_{h,x+\Delta x}} - 2\left(\frac{A_{x+\Delta x}}{A_x} - 1\right)$$



(b) Mach number range, 0.075 to 0.25.

Figure 2. - Continued. Evaluation of F(M,Λ) and G(M) (ratio of specific heats, 1.40).